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MEASURES OF AIRCRAFT EFFECTIVENESS

Richard H. Anderson
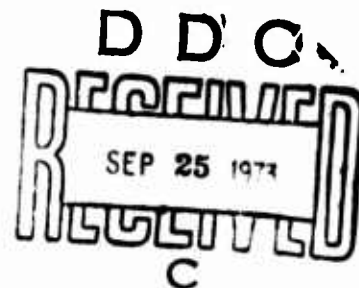
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OFFICE OF THE ASSISTANT FOR STUDY SUPPORT
DCS/Development Plans, Air Force Systems Command
Kirtland Air Force Base, New Mexico 87117

FOREWORD

This report presents various measures of effectiveness for the evaluation of aircraft. The need for such measures arose from OAS analysis efforts in life cycle cost and test and evaluation initiated at the request of DCS/Development Plans, Headquarters AFSC.

This report has been reviewed and is approved for publication.

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Technical Director

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ABSTRACT

In this report some overall measures of aircraft effectiveness are examined, e.g., targets destroyed after a given number of sorties, time rate of destruction, lifetime targets destroyed, and exchange ratio. Mathematical relationships are developed to show the dependence of the overall measures of effectiveness upon certain fundamental effectiveness parameters such as weapon delivery accuracy, survivability, reliability, and availability. A special section is devoted to measures of effectiveness of air-to-air fighters showing the dependence of the fighter exchange ratio upon such fundamental parameters as first shot capability and relative weapon effectiveness. Hopefully, the results will provide some illumination and guidance to planners and testers in identifying and assessing the relative importance of the fundamental effectiveness parameters and also in developing methods and techniques for evaluating these parameters during testing.

CONTENTS

1. INTRODUCTION

The utility of an aircraft (for instance, tactical interdiction aircraft) is dependent upon the kill potential (e.g., number of targets destroyed per successful sortie), the probability of reaching the target without an abort, the probability of survival, and the availability. The kill potential depends upon the number and type of weapons carried, acquisition probability, delivery accuracy, and target type. The probability of no abort is dependent upon the number, complexity, and reliability of subsystems. Survival probability is dependent upon the number and type of enemy defenses and such aircraft characteristics as ECM, radar cross section, IR signature, armor, and other protective measures. The availability of an aircraft depends upon the frequency of repairs and the average repair time (time to restore).

The worth of an aircraft cannot be assessed by considering any one of the above factors in isolation. All of the factors must be considered simultaneously to account for their interaction. The purpose of this paper is to develop measures of effectiveness of an aircraft which quantitatively account for the interaction of the characteristic effectiveness parameters.

For most mission types, an aircraft will be sent on repeated sorties provided it survives; thus, any valid measure of effectiveness must account for the cumulative effect of repeated sorties. It is also clear, and will be shown quantitatively, that survivability is of the utmost importance since it determines the average number of sorties an aircraft can complete.

If a particular scenario is specified, then for a given aircraft the characteristic effectiveness parameters serve to characterize that aircraft

and scenario. Since in "real life" the scenario changes sortie by sortie, the determination of aircraft performance over repeated sorties requires that the characteristic parameters be specified for each sortie. Such a detailed specification would introduce a high level of arbitrariness leading to an unsuitable measure of a system's worth. However, to obtain a measure (not a predictor) of the effectiveness of an aircraft in a given scenario it seems reasonable to keep the scenario fixed (fixed characteristic parameters) and to determine the cumulative effectiveness if the aircraft flew repeated sorties (when it survives) in that fixed scenario. This is the basic idea underlying the measures of effectiveness developed in this paper.

The first measure developed is the expected number of targets destroyed if the aircraft flies up to S sorties in a fixed scenario, i.e., aircraft flies repeated sorties (up to a maximum of S) if it survives. The next measure is the lifetime targets destroyed, i.e., aircraft flies repeated sorties as long as it survives. However, the expected number of targets destroyed during the lifetime of an aircraft may not be the prime measure of effectiveness since there are situations in which it is more important to know the effectiveness of an aircraft over, for instance, a 10 or 20 day period. The final measure of effectiveness developed is the expected number of targets destroyed as a function of time, which yields targets destroyed over any prescribed time period.

Although the discussion is in terms of tactical interdiction aircraft, the kill potential can be redefined (for example, in terms of cargo tonnage delivered or enemy aircraft destroyed) to account for airlift, counterair, or other type aircraft. A special section is devoted to the measures of

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effectiveness for air-to-air fighters. It is shown that the effectiveness of a fighter aircraft is strongly dependent upon first shot capability since the first shot has a major effect on both the kill potential and the survival probability of a fighter.

2. LIFETIME DESTRUCTION

The definitions listed below will facilitate the mathematical developments contained in this section.

P_{s1} = P {aircraft survives to release its weapons on target}.

P_{s2} = P {aircraft survives return trip after weapons are released}.

P_c = P {aircraft reaches target and releases weapons without an abort causing failure given that it survives}.

P_{sa} = P {aircraft aborts before releasing weapons and survives the return trip}.

ρ = "Kill Potential" = expected number of targets destroyed after aircraft reaches the target area.

P_s = Single sortie survival probability.

S = Number of sorties aircraft flies (if it survives).

T (S) = Expected number of targets destroyed after S sorties .

T = Expected number of targets destroyed during the "lifetime" of the aircraft, i.e., $S \to \infty$.

For a single sortie, the expected number of targets destroyed by an aircraft is

$$T(1) = \rho \, P_c \, P_{s1} . \tag{1}$$

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The main problem in this section is to determine the expected number of targets destroyed if the aircraft flies a maximum of S sorties (if it survives). The time required to complete S sorties is treated in the next section. The probability P_i that the aircraft starts its i-th ortie (i $\leq$ S) is equivalent to the probability it survives the first i-1 sorties. Therefore,

$$P_i = \{P_{s1} P_c P_{s2} + P_{sa}\}^{i-1} = P_s^{\,i-1} \qquad (i = 1, 2, \ldots, S), \qquad (2)$$

where P_s denotes the single sortie survival probability. The expected damage from the i-th sortie is

$$P_i \, \rho \, P_c \, P_{s1} = P_s^{\,i-1} \, \rho \, P_c \, P_{s1} . \qquad (3)$$

Therefore, it follows that the expected number of targets destroyed after S sorties (with a fixed scenario) is

$$T(S) = \sum_{i=1}^{S} P_i \, \rho \, P_c \, P_{s1}$$

$$= \rho \, P_c \, P_{s1} \sum_{i=1}^{S} P_s^{\,i-1} = \rho \, P_c \, P_{s1} \left\{ \frac{1 - P_s^{S}}{1 - P_s} \right\} . \qquad (4)$$

Letting $S \to \infty$ in equation (4) it follows that the expected number of targets destroyed during the lifetime of the aircraft is

$$T = \frac{\rho \, P_c \, P_{s1}}{1 - P_s} . \qquad (5)$$

Of course, if for any reason there is an upper limit to the number of sorties the aircraft would fly, then this number should be used in equation (4) to determine the expected damage during the useful lifetime of the aircraft.

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The expression (5) for lifetime destruction was derived under the assumptions that the aircraft flies repeated sorties as long as it survives and that the scenario remains the same for each sortie. It is important to point out that this measure of effectiveness has another interpretation. Suppose N (N = 1, 2, 3, ...) aircraft each fly one sortie where the parameters ρ, P_c, P_{s1}, and P_s are the same for each aircraft. The expected number of targets destroyed by the N aircraft is

$$N \rho P_c P_{s1} \cdot \tag{6}$$

The expected number of aircraft lost is

$$N (1 - P_s) \cdot \tag{7}$$

The ratio of the quantities (6) and (7) yields a measure of targets destroyed per aircraft lost (exchange ratio) equal to

$$\frac{\rho P_c P_{s1}}{1 - P_s} \tag{8}$$

which is independent of the number of aircraft. This exchange ratio is identical to expression (5) for lifetime targets killed.

The expected number of sorties completed during the lifetime of an aircraft is

$$\langle S \rangle = \sum_{j=1}^{\infty} j P_s^j (1 - P_s) = \frac{P_s}{1 - P_s} \cdot \tag{9}$$

This measure is further discussed in Section 4.

Although the measures (4) and (5) are useful indicators of the effectiveness of an aircraft, they do not reflect the time rate of damage.

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This is the subject of the next section.

3. TARGETS KILLED AS FUNCTION OF TIME

Equation (4) gives the expected number of targets destroyed after S sorties. However, in evaluating the effectiveness of an aircraft, it is also essential to determine the expected time required for the S sorties. This time depends, of course, upon the mission time T_m and also upon the time required to make repairs.

If the aircraft completes S sorties then the expected number of repairs is

$$\frac{S\ T_m}{\tau}\ ,\tag{10}$$

where τ is the MTBF of the total aircraft system.
Therefore, the expected total repair time is

$$\frac{S\ T_m}{\tau}\ t_r\ ,\tag{11}$$

where t_r is the mean time to restore. If Δt denotes the average time for normal service actions, e.g., refuel and reload, then the expected time to complete S sorties is

$$t(S)\ =\ S\left\{T_m + \frac{T_m}{\tau}\ t_r + \Delta t\right\}\ .\tag{12}$$

If the service actions can be performed while repairs are being made, then Δt in equation (12) should be replaced by

$$\min\left\{\Delta t - \frac{T_m}{\tau}\ t_r,\ 0\right\}\ .\tag{13}$$

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In Handbook Reliability Engineering (NAVAIR 00-65-502) availability is defined as

$$A = \frac{1}{1 + t_{r/\tau}} .$$ (14)

From this it follows that

$$\frac{t_r}{\tau} = \frac{1}{A} - 1 .$$ (15)

Therefore, equation (12) becomes

$$t(S) = S \left\{ \frac{T_m}{A} + \Delta t \right\} .$$ (16)

Equations (4) and (16) provide the expected number of targets destroyed as a function of time.

4. EXAMPLES

a Lifetime Sorties

Figure 1 shows the expected number of sorties completed during the lifetime of an aircraft as a function of survival probability. Since the lifetime targets killed T is a constant factor multiplied by lifetime sorties, the curve for T has the same shape as the curve in Figure 1. Of course, the curve cannot be extended indefinitely since there is an upper limit based upon the service life of the aircraft or other such factors. Several conclusions are apparent:

(1) Conditions resulting in survival probabilities below .95 are probably unacceptable in most cases since lifetime sorties is less than 19.

(2) Small improvements in survival probability in the region $P_s < .98$ result in a small increase in lifetime sorties. However, in the region of
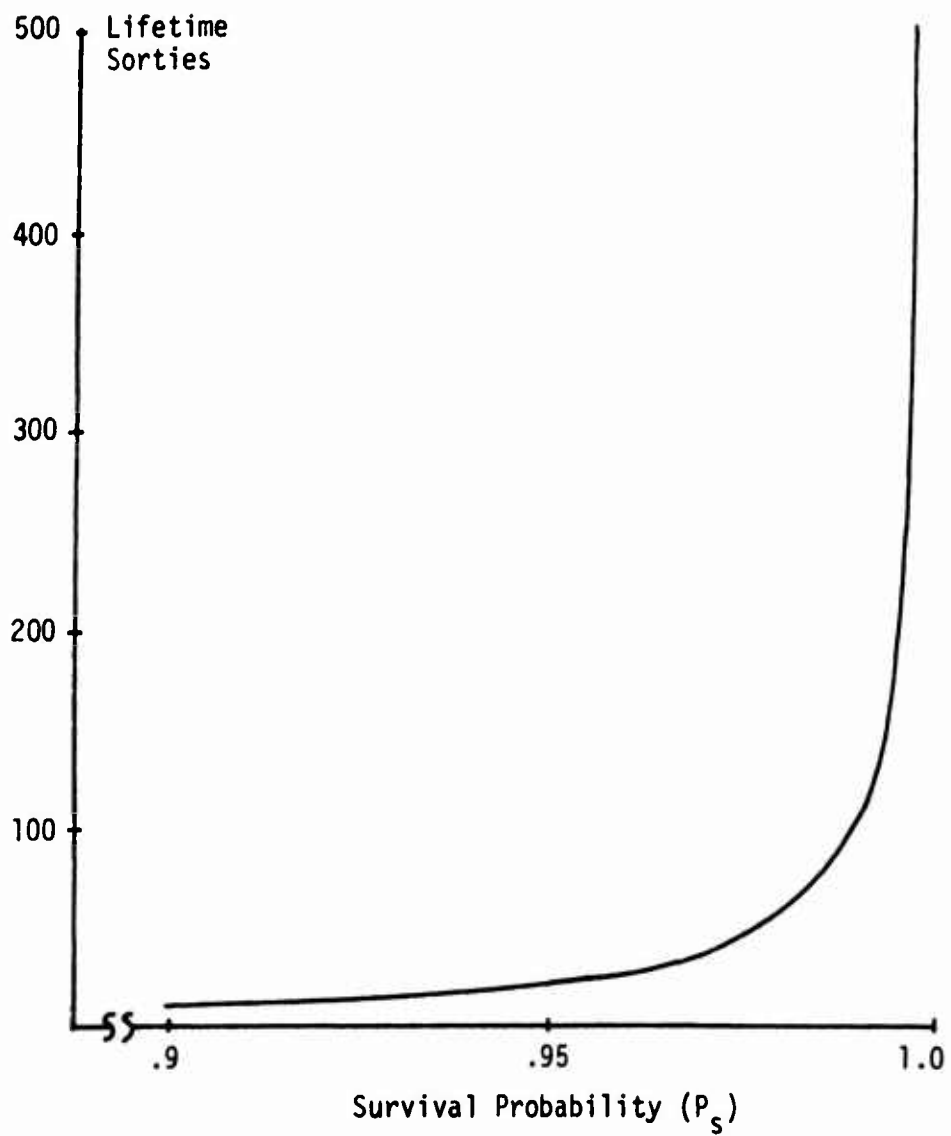
Figure 1. Lifetime Sorties as Function of Survival Probability

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high P_s (e.g., $P_s > .98$) any small increase in P_s results in a dramatic increase in lifetime sorties. For example, the small increase in P_s from .990 to .995 more than doubles the number of lifetime sorties (from 99 to 199).

(3) Survival probability can be, by far, the most dominant factor in determining the effectiveness of aircraft.

To get a feeling for the magnitude of the numbers involved, it is instructive to consider a historical but recent engagement in a rather severe environment where U.S. aircraft flew 1000 sorties against heavily defended targets. During this period, 26 U.S. aircraft were lost. The survival probability in this case was $P_s = 0.974$ which is on the low part of the curve in Figure 1. Under such conditions the average number of sorties per aircraft is only 37.5.

One obvious means to increase survival probability is to reduce the enemy's defenses (gain air superiority). Survival probability can also be improved by designing the aircraft to reduce the probability of hit (e.g., ECM or reducing radar cross section and IR signature) and to reduce the probability of kill given the aircraft is hit (e.g., armor, foam in fuel tanks). Such methods can result in dramatic payoffs.

b. Comparing Aircraft

Table I shows the effectiveness parameters associated with 5 hypothetical aircraft labeled A, B, C, D, and E. Although each parameter is important in the evaluation of an aircraft, it appears impossible to rank the 5 aircraft by studying the table. The table does show that aircraft A has the best kill potential, B has the highest probability of reaching

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the target without an abort, C has the highest survival probability, and D has the highest availability.

Table I

EFFECTIVENESS PARAMETERS FOR FIVE AIRCRAFT

EFFECTIVENESS PARAMETERS	A I R C R A F T T Y P E				
	A	B	C	D	E
Kill Potential (e.g., targets killed per successful sortie)	2.5	1.8	.80	2.0	2.1
P_c (Probability of reaching target without an abort)	.90	.93	.90	.82	.85
P_s (Survival Probability)	.970	.990	.999	.980	.995
A (Availability)	.85	.87	.83	.90	.83
Mission Time (hr)	2	2	2	2	2
Service Time (hr)	.5	.5	.5	.5	.5

Using the parameters listed in Table I together with equations (4) and (16), the expected number of targets destroyed as a function of time can be calculated for each aircraft. The results in Figure 2 are based upon continuous operation, i.e., aircraft is launched as soon as it is ready. Although aircraft E does not dominate the others in any of the effectiveness parameters, when all parameters are integrated aircraft E is superior to the others (E and A are about equal in the beginning) at least for time less than 150 hours (about 51 missions). The lifetime targets destroyed by each aircraft indicates where the curves finally level off. The lifetime targets destroyed (LTD) by each aircraft are:
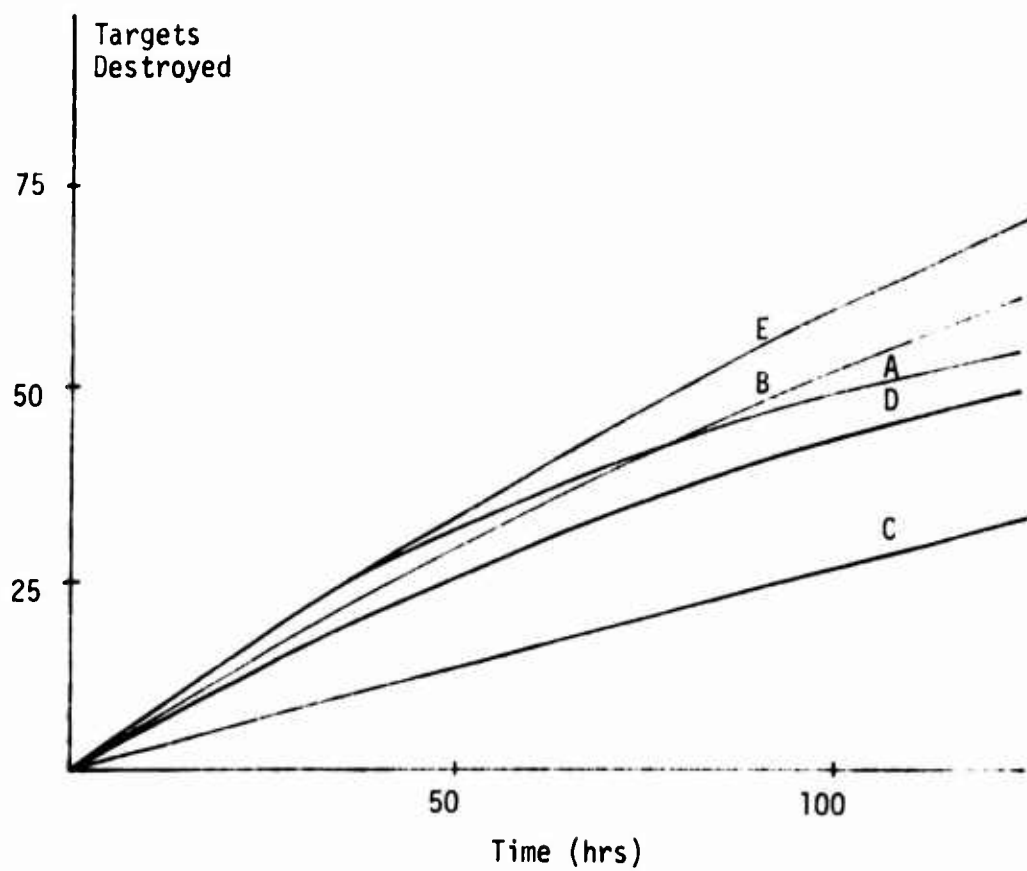
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Figure 2.   Destruction as Function of Time for Five Hypothetical
            Aircraft

A: LTD = 73

B: LTD = 166

C: LTD = 719

D: LTD = 80

E: LTD = 355.

This indicates that C might be better than E since its curve will eventually rise above the targets destroyed curve of aircraft E. Figure 3 shows targets destroyed by C, E, and B as a function of time when time is carried out to 2500 hours (about 859 missions). Although aircraft C and E are the only two competitors, aircraft B is shown merely to demonstrate that its low survivability causes its curve to level off early at a LTD of 166. Figure 3 shows that E is substantially better than C for time less than 1830 hours (629 missions). For times greater than this the higher survivability of C more than compensates for its lower kill potential and C is better than E. The analysis shows that E is better than aircraft A, B, and D. However, the selection between C (with a higher LTD) and E is dependent upon the preference of the decision-maker, i.e., whether short term or long term performance is of prime interest.

5. AIR-TO-AIR FIGHTERS (IMPORTANCE OF FIRST SHOT)

It is particularly interesting to apply some of the ideas of the previous sections to air-to-air engagements between fighter aircraft. It is intuitively clear that the probability of maneuvering into position to fire the first shot is an important factor in determining the effectiveness of a fighter aircraft. This is due to the fact that the first shot probability has a strong influence on both the kill potential and the survival probability of the fighter. The tools developed in the previous
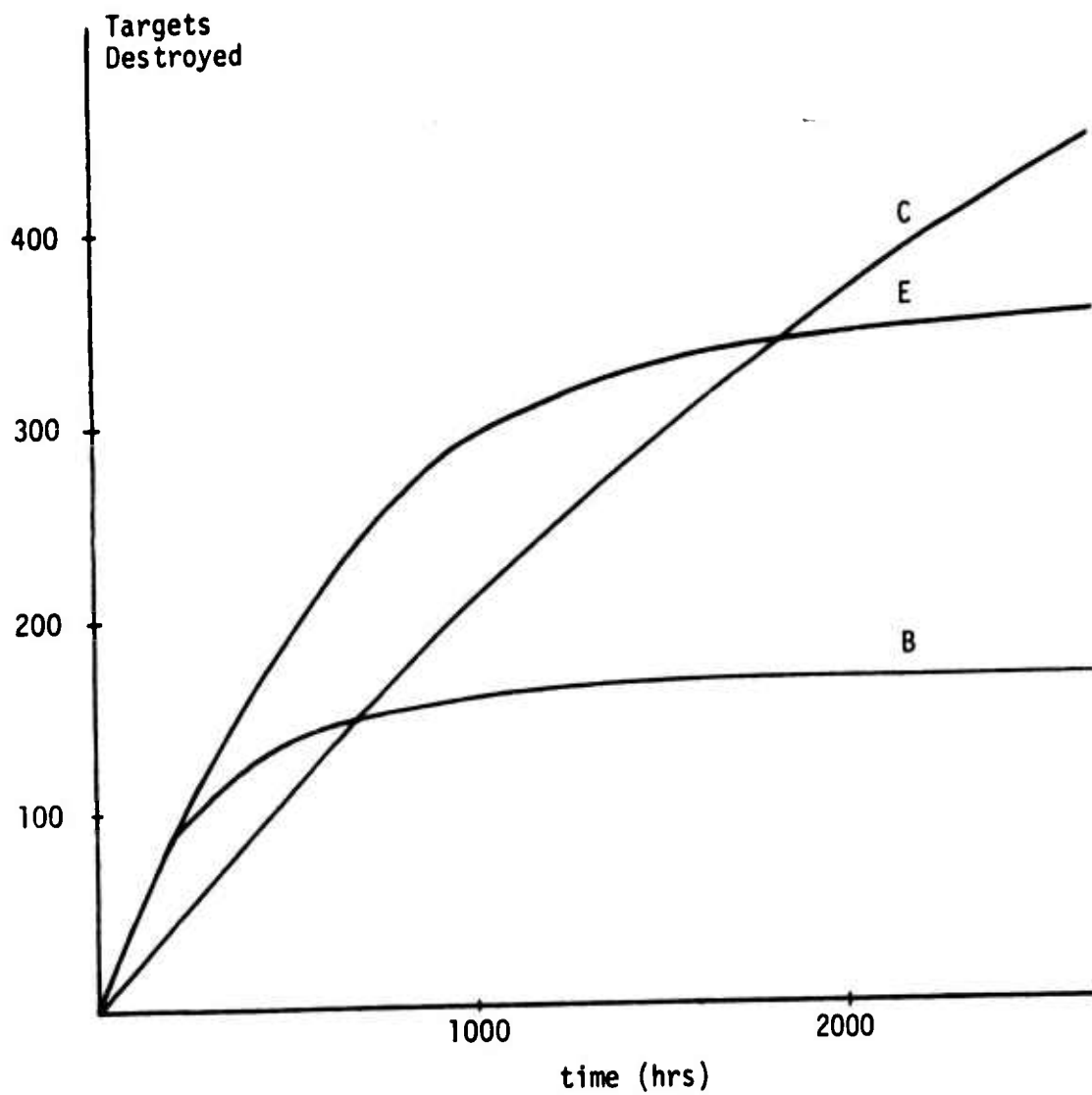
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Figure 3.   Destruction as Function of Time for Aircraft B, C, and E

sections provide a means to show quantitatively the influence of first
shot probability on the exchange ratio (i.e., Red fighters destroyed
per Blue fighter destroyed). This exchange ratio can also be interpreted
as the expected number of enemy fighters destroyed during the lifetime of
a Blue fighter. The first air-to-air scenario is described in the next
paragraph.

In an air-to-air engagement between a Blue and a Red fighter, the
probability that the Blue fighter fires the first shot is denoted by P_1.
This first shot probability is a function of acquisition and tracking
capabilities, speed, maneuverability, and pilot skills. The fighter firing
the first shot releases its air-to-air weapons destroying the other
fighter with a certain probability (P_{kb} for Blue weapons, P_{kr} for Red
weapons). If the attacked fighter is destroyed the engagement is fin-
ished; however, for this first scenario it is assumed that if the attacked
fighter is not destroyed it maneuvers into position to launch its weapons
against the other aircraft (this assumption will be modified later). The
engagement is then finished with each fighter getting at most one pass.
Although multiple passes could easily be considered, it requires additional
assumptions and contributes little to the understanding of the problem
(especially with effective air-to-air weapons).

The first quantity to be derived is the probability that the Blue
fighter destroys the Red fighter in a given engagement. This is the
fighter kill potential; it is equal to the probability that the Blue fighter
fires the first shot and destroys the Red fighter plus the probability that the
Red fighter fires first shot and misses the Blue fighter and the Blue fighter

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then destroys the Red fighter. Thus,

$$\rho = P_1 \, P_{kb} + (1 - P_1) \, (1 - P_{kr}) \, P_{kb}$$

$$= P_{kb} \, \{1 - P_{kr} \, (1 - P_1)\}. \tag{17}$$

where P_1 denotes the first shot probability of the Blue fighter, P_{kb} is the kill probability of the weapons of the Blue fighter, and P_{kr} is the kill probability of Red weapons.

The next expression to be derived is the single engagement survival probability P_s of the Blue fighter. The Red fighter will be prevented from launching its weapons if the Blue fighter fires the first shot and destroys the Red fighter. Therefore, the probability that Red attacks the Blue fighter is

$$(1 - P_1 \, P_{kb}) \, . \tag{18}$$

The survival probabiiity is then

$$P_s = 1 - P_{kr} \, (1 - P_1 \, P_{kb}). \tag{19}$$

From equations (17) and (19) it follows that the exchange ratio is given by

$$ER = \frac{\rho}{1 - P_s} = \frac{P_{kb} \, \{1 - P_{kr} \, (1 - P_1)\}}{P_{kr} \, (1 - P_1 \, P_{kb})} \, . \tag{20}$$

Figure 4 is presented to illustrate the strong influence of first shot probability on both the probability of survival of the Blue fighter and the probability of survival of the Red fighter. In this example the
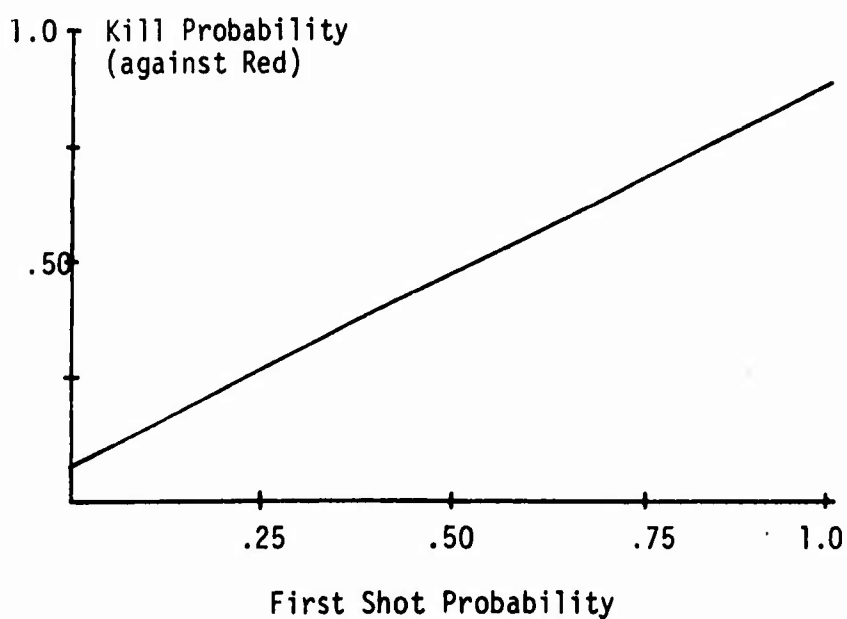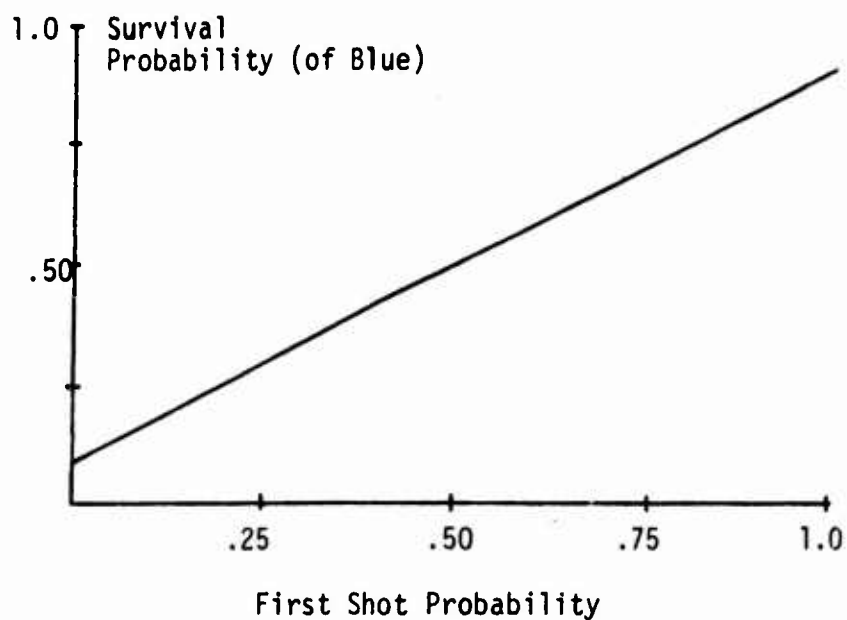
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Figure 4. The Effect of First Shot Probability on Survival and Kill Probability ($P_{kb} = P_{kr} = 0.9$)

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effectiveness of Red and Blue weapons is assumed to be equal, i.e., $P_{kb} = P_{kr} = 0.9$. Although weapons are equally effective, the first shot capability of Blue can cause the survival probability to vary from 0.10 to 0.91 and the kill probability against the Red fighter to vary from 0.09 to 0.90.

Figure 5 incorporates both the kill potential and survival probability to show the dependence of the exchange ratio upon the first shot capability. Two cases are presented corresponding to Red weapon effectiveness of $P_{kr} = 0.6$ and $P_{kr} = 0.9$. For each case the exchange ratio is plotted for $P_{kb} = 0.6$ and $P_{kb} = 0.9$. Several conclusions are apparent:

a. Effective Blue weapons and a high first shot capability are both necessary for achievement of a high exchange ratio for Blue.

b. Even when $P_{kb} = 0.9$ and $P_{kr} = 0.6$ a first shot probability below $P_1 = 0.22$ results in an exchange ratio below 1.0, i.e., the advantage of a superior weapon can be nullified by a poor first shot capability.

c. The disadvantage of a poor weapon (e.g., $P_{kb} = 0.6$ and $P_{kr} = 0.9$) can sometimes be more than compensated for by a high first shot capability.

In the previous scenario it was assumed that whenever the fighter firing the first shot missed, the other fighter then maneuvered into position to fire its weapons. However, a fighter may fire the first shot and miss but still have the capability to outmaneuver the other fighter thereby avoiding being fired upon. To account for this, the following probabilities are introduced:

P_{mb} P {Blue fighter avoids being fired upon whenever it fires first shot and misses}
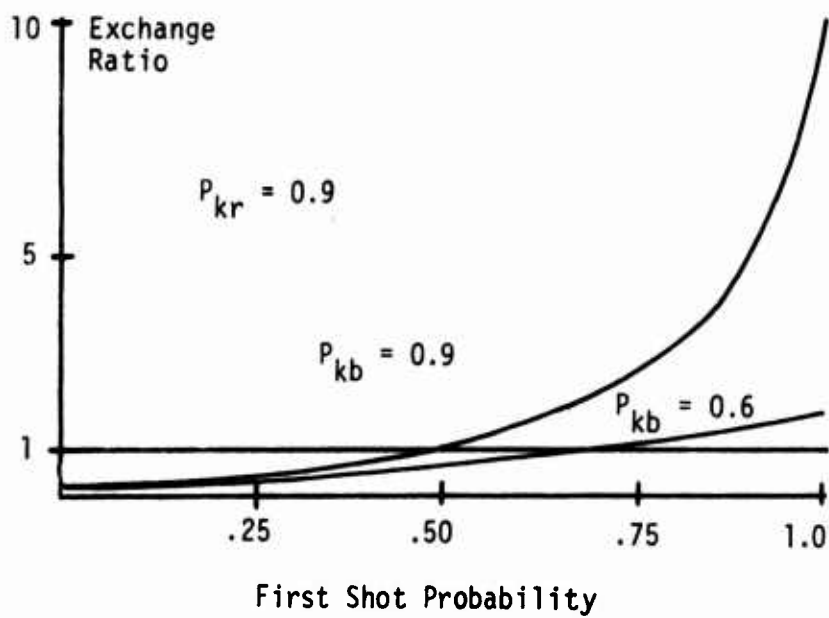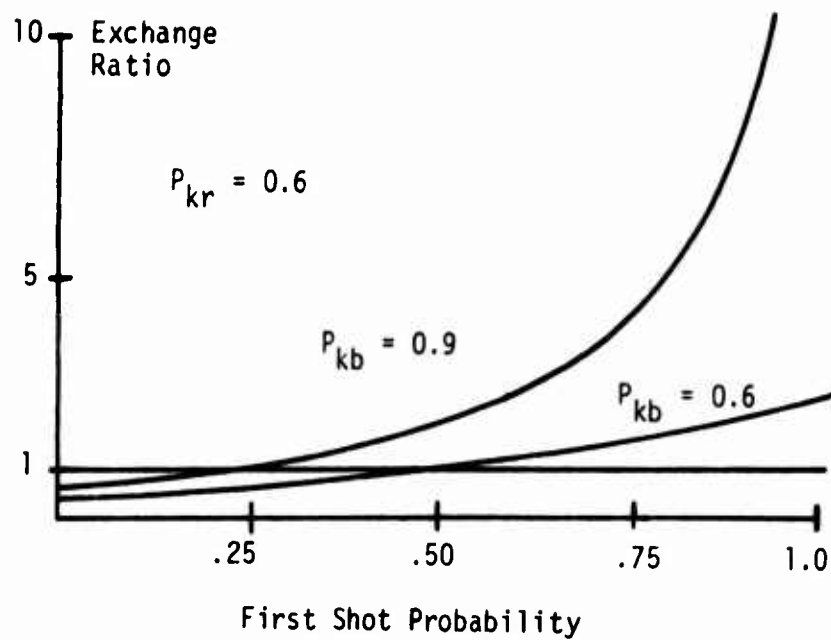
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Figure 5. Exchange Ratio as Function of First Shot Probability

$P_{mr} \equiv P$ {Red fighter avoids being fired upon whenever it fires first shot and misses}

The probability that the Red fighter is destroyed becomes

$$\rho = P_{kb} \{P_1 + (1 - P_1) (1 - P_{kr}) (1 - P_{mr})\} . \qquad (21)$$

The probability that the Blue fighter is destroyed is

$$1 - P_s = P_1(1 - P_{kb}) (1 - P_{mb}) P_{kr} + (1 - P_1) P_{kr}$$

$$= P_{kr} \{1 - P_1 (P_{kb} + P_m - P_m P_{kb})\} . \qquad (22)$$

From equations (21) and (22) it follows that the exchange ratio is

$$ER = \frac{P_{kb} \{P_1 + (1 - P_1) (1 - P_{kr}) (1 - P_{mr})\}}{P_{kr} \{1 - P_1 (P_{kb} + P_{mb} - P_{mb} P_{kb})\}} . \qquad (23)$$

For $P_{mb} = P_{mr} = 0$, equation (23) reduces to equation (20).

The most favorable case for Blue is when $P_{mb} = 1$ and $P_{mr} = 0$; the most unfavorable case is $P_{mb} = 0$ and $P_{mr} = 1$. Using these extreme cases, the bounds for the exchange ratio are shown in Figure 6 where the solid curves are identical to those in Figure 5, i.e., $P_{mb} = P_{mr} = 0$. As seen by comparing the solid and lower curves in the figures, if Red can out-maneuver Blue after getting first shot but Blue does not have this capability ($P_{mb} = 0$, $P_{mr} = 1$) this has little effect on the exchange ratio since it has no effect on Blue's survival probability. However, if $P_{mb} = 1$ and $P_{mr} = 0$, Blue's survival probability is improved, and hence the exchange ratio is improved significantly if the first shot probability is high; furthermore, the lower the value of P_{kb} the greater the importance of the capability of Blue being able to outmaneuver Red after firing the first shot.
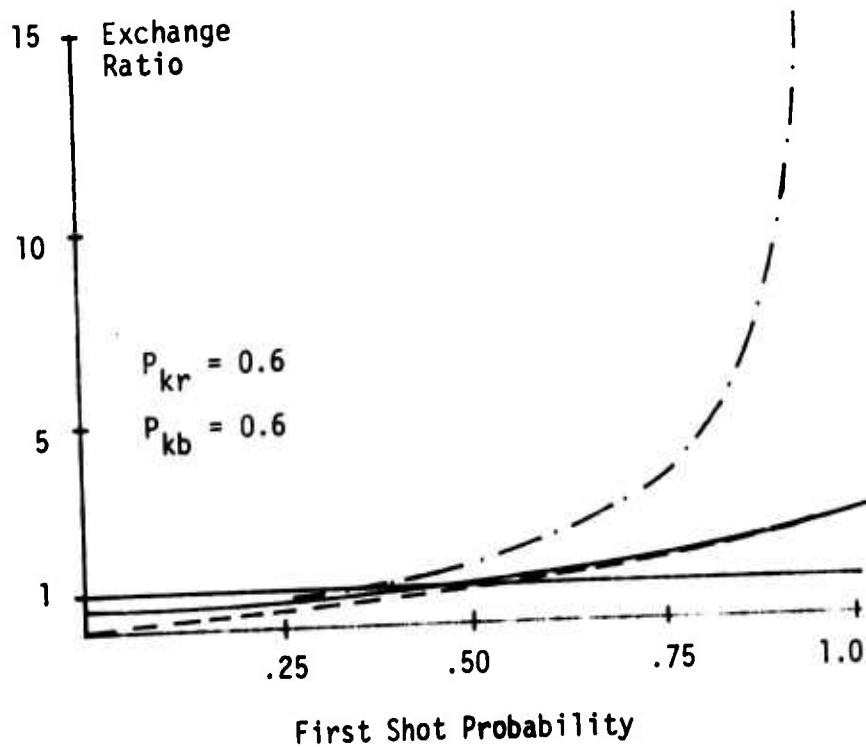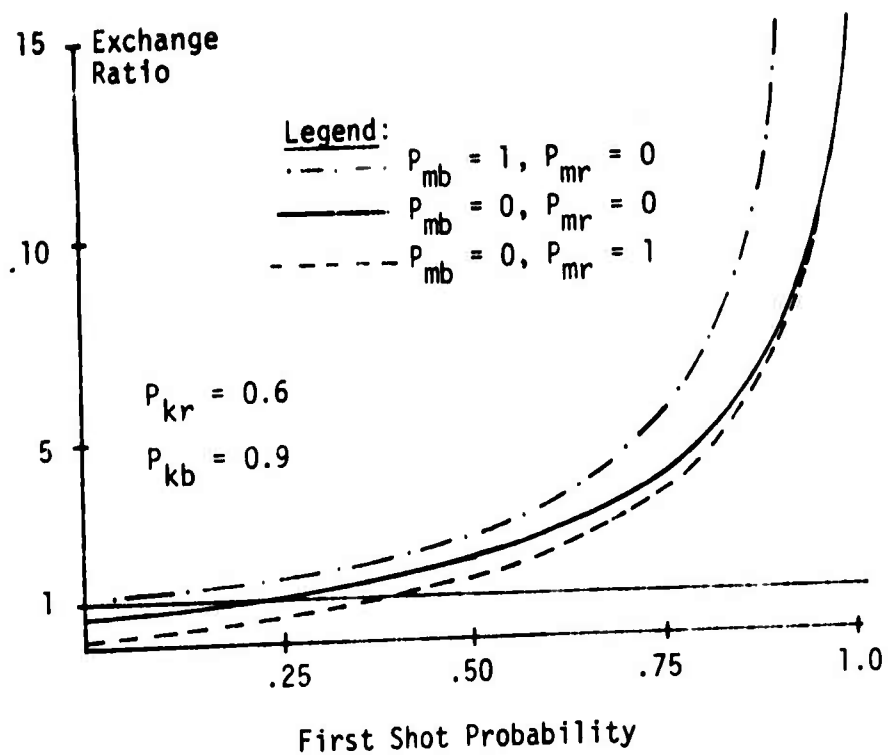
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Figure 6. Importance of Maneuverability After
First Shot ($P_{kr} = 0.6$)

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6. CONCLUSIONS

a. An evaluation of the effectiveness of an aircraft must account for the interaction of kill potential, abort probability, survival probability, and availability. Individually, these characteristic parameters do not determine the worth of an aircraft.

b. Any valid measure of effectiveness must also account for the cumulative effect of repeated sorties.

c. The measures of effectiveness developed in this paper provide a simple means of integrating the characteristic effectiveness parameters to determine the cumulative damage accrued by repeated sorties.

d. Survival probability can be the most dominant factor in determining the lifetime effectiveness of an aircraft. For example, a 5% increase in kill potential results in a 5% increase in lifetime damage; however, a 5% increase in survival probability, say from P_s = .95 to P_s = .9975, results in a 2100% increase in lifetime damage.

e. Since survivability is of such great importance it warrants special emphasis during design and testing. Survival probability is an extremely important factor in comparing two aircraft; for instance, one aircraft may have a poorer weapon delivery accuracy and yet be far superior because of higher survivability.

f. In the case of air-to-air combat, the exchange ratio (Red fighters destroyed per Blue fighters destroyed) is an important measure of worth.

g. The exchange ratio for a fighter aircraft can be expressed as a function of three fundamental parameters: weapon effectiveness, first shot probability, and the capability to maneuver away (avoid being fired upon) after firing first and missing.

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h. The most important parameter affecting the exchange ratio is the first shot probability. The advantage of a superior weapon can be nullified by a poor first shot capability; and, conversely, the disadvantage of an inferior weapon can sometimes be compensated for by a good first shot capability.

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13. ABSTRACT

This report presents some measures of aircraft effectiveness expressed in terms of characteristic aircraft parameters. Examples are given to show the importance of aircraft survival and the importance of first shot capability for air-to-air fighter aircraft.

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Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
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